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THE USE OF OBSERVATION WELLS WITH SLUG TESTS

by

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Abstract

Slug tests are commonly used for site characterization. An earlier paper (EOS, v.70, no. 43, p.1078) dealt with the use of sensitivity analysis to design a test which would give reasonably accurate estimates of the aquifer parameters by an informed choice of the number and times of measurements. Most practitioners know that slug tests are not very sensitive to the storage coefficient, as explained in the earlier paper. An investigation of the radial dependence of the Cooper et al. (1967) analytical slug test solution shows that the use of one or more observation wells can vastly improve the parameter estimates, particularly the estimate for storage. While it would usually not be practical to install an observation well solely for use in a slug test, many times nearby wells are available. Generally, the observation well must be fairly close (a few tens of feet or less) to the slugged well to be effective. The storage coefficient must be small in order to see the effect of the slug at greater distances from the slugged well. Since the temporal and spatial dependence of the sensitivities for transmissivity and storage are considerably different, the addition of one or more observation wells will substantially reduce the correlation between these two parame-ters, and result in much better estimates than usually obtained in slug tests. These ideas are illustrated using typical data from our research sites.

The Use of Observation Wells With Slug Tests

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Introduction

Slug tests are commonly used for site characterization. An earlier paper (EOS, v.70, no. 43, p.1078) dealt with the use of sensitivity analysis to design a test which would give reasonably accurate estimates of the aquifer parameters by an informed choice of the number and times of measurements. Most practitioners know that slug tests are not very sensitive to the storage coefficient, as explained in the earlier paper. An investigation of the radial dependence of the Cooper et al. (1967) analytical slug test solution shows that the use of one or more observation wells can vastly improve the parameter estimates, particularly the estimate for storage.

Cooper - Bredehoeft - Papadopulos Analytical Solution for Slug Tests

$$H\left(\alpha, \beta, H_o, \frac{r}{r_s}\right) = \frac{8H_o\alpha}{\pi^2} \int_0^\infty \frac{e^{-\beta u^2/\alpha}}{u\Delta(u)} F\left(u, \alpha, \frac{r}{r_s}\right) du$$

$$F\left(u, \alpha, \frac{r}{r_s}\right) = \left\{ J_0\left(u\frac{r}{r_s}\right) \cdot \left[uY_0(u) - 2\alpha Y_1(u)\right] - Y_0\left(u\frac{r}{r_s}\right) \cdot \left[uJ_0(u) - 2\alpha J_1(u)\right] \right\}$$

$$\Delta(u) = \left[uJ_0(u) - 2\alpha J_1(u)\right]^2 + \left[uY_0(u) - 2\alpha Y_1(u)\right]^2$$

$\beta = \frac{Tt}{r_c^2}$ J, Y are Bessel Functions

$\alpha = \frac{r_s^2}{r_c^2} S$ r_s and r_c are screen and casing radii

$\frac{H}{H_0} = h \equiv$ relative head

$H_0 \equiv$ initial head displacement

Slug-test responses can be expressed as a function of four parameters: alpha, a parameter related to screen and casing radii and the storage coefficient; beta, a dimensionless time involving transmissivity and the casing radius; H_0, the initial head displacement; and r/r_s, the distance to an observation well divided by the screen radius.

SENSITIVITY COEFFICIENTS

FIRST ORDER TAYLOR EXPANSION FOR THE HEAD

$$\mathbf{H}^{*} \cong \mathbf{H}^{m} + \mathbf{U}_{T}^{m}\Delta T^{m} + \mathbf{U}_{S}^{m}\Delta S^{m} + \mathbf{U}_{H_0}^{m}\Delta H_{0}^{m}$$

$\mathbf{H}^{*}$: vector of heads based on true parameters T^{*}, S^{*}, H_0^{*}

$\mathbf{H}^{m}$: vector of heads based on current estimates T^{m}, S^{m}, H_0^{m}

$\mathbf{U}_{T}^{m} = \dfrac{\partial \mathbf{H}^{m}}{\partial T^{m}}$, $\mathbf{U}_{S}^{m} = \dfrac{\partial \mathbf{H}^{m}}{\partial S^{m}}$, $\mathbf{U}_{H_0}^{m} = \dfrac{\partial \mathbf{H}^{m}}{\partial H_0^{m}}$: sensitivities to T^{m}, S^{m}, H_0^{m}

ΔT^{m}, ΔS^{m}, ΔH_0^{m} : unknown perturbations in transmissivity, storage coefficient, and initial head

NORMALIZED SENSITIVITIES TO RELATIVE HEAD

$$U'_T = T\frac{\partial H}{\partial T} = H_0 T\frac{\partial h}{\partial T} = H_0 u'_T$$

$$u'_T = T\frac{\partial h}{\partial T} = \beta\frac{\partial h}{\partial \beta}$$

$$U'_S = S\frac{\partial H}{\partial S} = H_0\, S\frac{\partial h}{\partial S} = H_0\, u'_S$$

$$u'_s = S\frac{\partial h}{\partial S} = \alpha\frac{\partial h}{\partial \alpha}$$

$$U'_{H_0} = H_0\frac{\partial H}{\partial H_0} = H_0\, h = H = H_0\, u'_{H_0}$$

$$u'_{H_0} = h$$

u'_T, u'_s, and u'_{H_0} are functions of α, β, and r/r_s

We shall look at these functions in greater detail later.

PARAMETER ESTIMATION

OBJECTIVE: Minimize $E = \sum_i \left[H_i^e - H_i \right]^2$

H_i^e = observed head at index point i

H_i = calculated head at index point i

ASSUME: $\mathbf{H}^e = \mathbf{H}^* + \varepsilon$

ε = error vector

THUS: $\mathbf{H}^e - \mathbf{H}^m = \mathbf{U}_T^m \Delta T^m + \mathbf{U}_S^m \Delta S^m + \mathbf{U}_{H_0}^m \Delta H_0^m + \varepsilon$

SOLVE: New parameter estimates

$T^{m+1} = T^m + \Delta T^m \qquad S^{m+1} = S^m + \Delta S^m \qquad H_0^{m+1} = H_0^m + \Delta H_0^m$

SENSITIVITY DESIGN MATRIX

$$a_{ij} = [A]_{ij} = \sum_{r,t} U_i(\frac{r}{r_s}, t) U_j(\frac{r}{r_s}, t)$$

$$i, j = H_0, T, S$$

The sensitivity design matrix [A] defined here is a sum over time and space of products for any two sensitivity coefficients. If we are fitting all three parameters H_0, T, and S then the sensitivity design matrix is 3 x 3. The least squares solution for the delta parameter changes can be expressed in terms of the inverse of [A]. In general, the solution is well behaved if the diagonal elements are large and nearly equal and the off-diagonal elements are small. This will be the case if the sensitivity coefficients are large and do not have similar shapes over the measurement times and locations.

SENSITIVITY CORRELATION MATRIX

$$c_{ij} = \left[C\right]_{ij} = \frac{a_{ij}}{\sqrt{a_{ii}a_{jj}}}$$

One way to measure the similarity of the sensitivity coefficients is to define the sensitivity correlation matrix as shown here; it will have ones on the diagonal and the off-diagonal terms will vary between ± 1. If any of the off-diagonal terms are exactly one, the inverse of [A] does not exist and the inverse problem can not be solved for aquifer parameters. From a practical standpoint, anytime the off-diagonal elements of the correlation matrix get above .9 the [A] matrix becomes ill-conditioned rather rapidly and the inverse solution becomes more unreliable.

PARAMETER COVARIANCE MATRIX

$$\mathrm{COV}\,(P) = [B] = [A]^{-1}\sigma^2$$

σ = head variance

Estimated Standard Error
of Parameter i

$$\sqrt{b_{ii}}$$

As long as an inverse of [A] can be found, the reliability of the parameter estimates can be assessed by looking at the parameter covariance matrix defined here. The form shown here results from some simplifying assumptions about the errors in head such as additive, zero mean, noncorrelated and constant variance. With these assumptions the estimated standard errors of the parameters are given by the square roots of the diagonal elements of the parameter covariance matrix.

Relative Head or Sensitivity to H_0

Figure 1 shows the relative head or sensitivity to H_0, versus dimensionless time for various radii. The maximum occurs for $r = r_s$ (Figures 1 and 3), and decreases with time and distance. Figure 3 shows that the area of influence spreads with time. Figure 1 shows that an observation well will respond in time with a bell shaped curve whose maximum amplitude decays with distance from the slugged well. At about $175r_s$ the response has fallen to about .01 H_0 at a dimensionless time of 10 for $\alpha = 10^{-3}$ (Figures 1 and 2). Smaller α's result in larger responses in space and time as shown in Figures 2 and 4.

Figure 1.
Variation of u'_{H_s} With Time For Various r/r_s

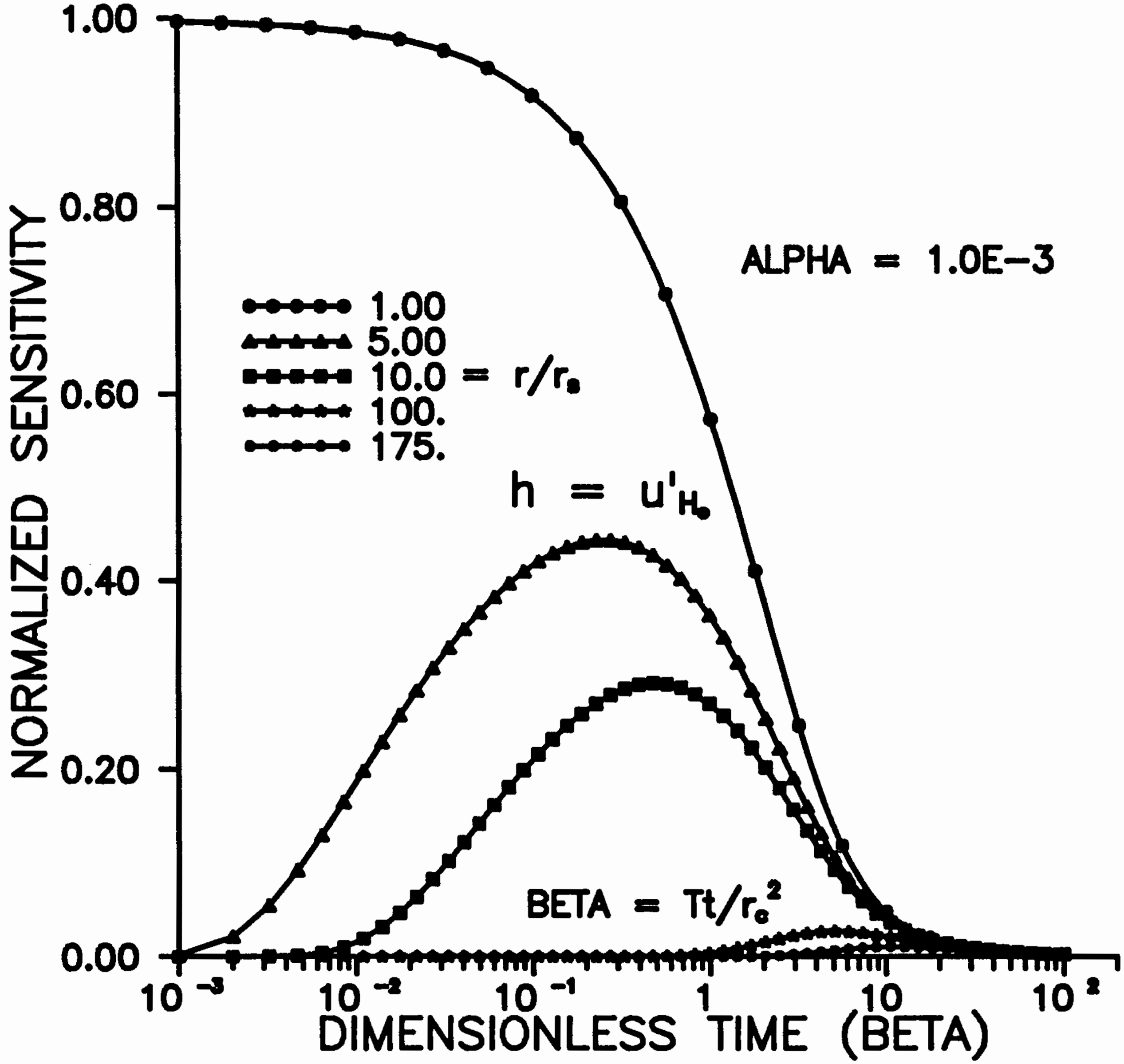


Figure 2.
Variation of u'_{H_e} With Time For Various Alpha

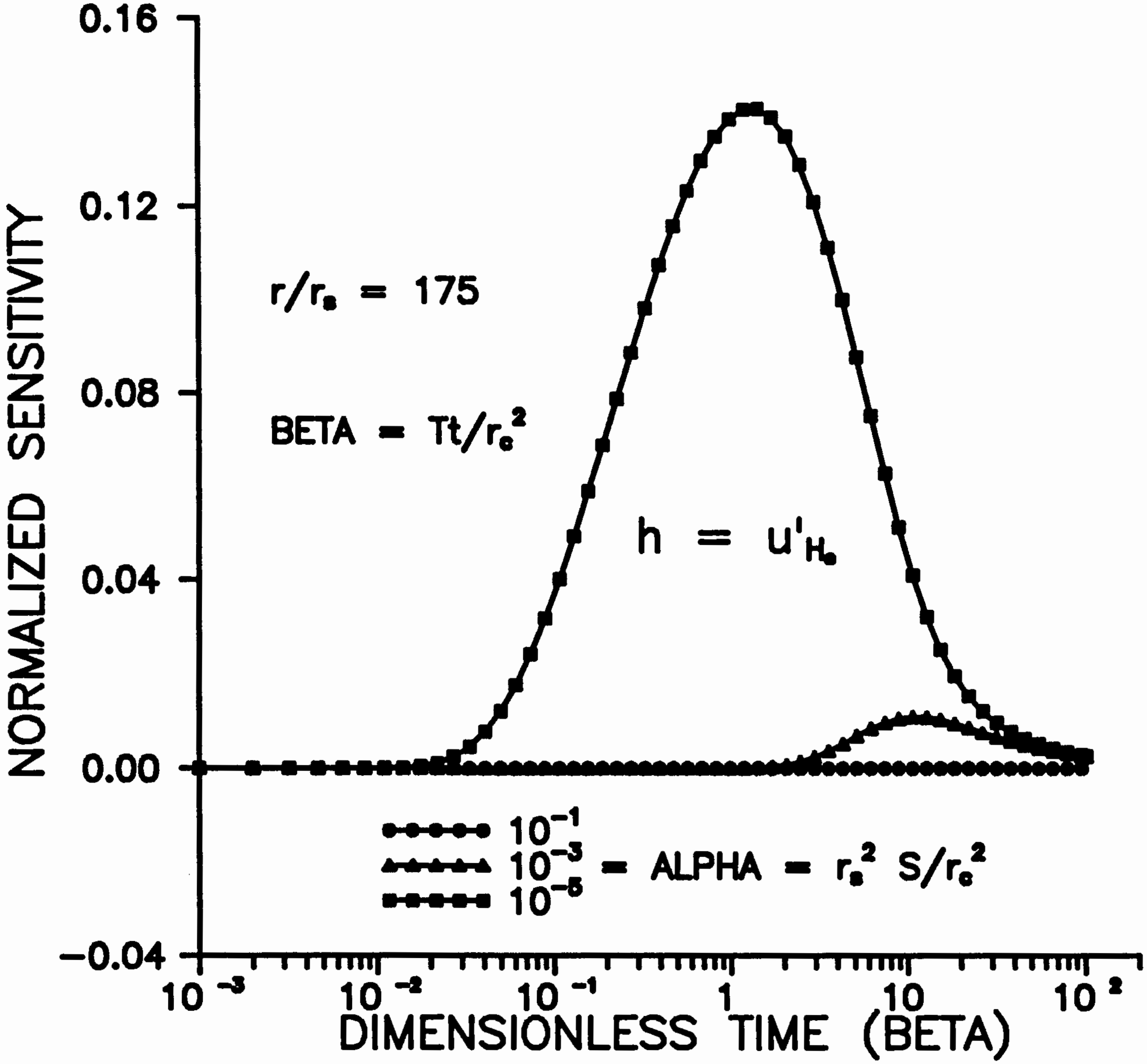


Figure 3.
Variation of u'_{H_0} With Distance For Various Times

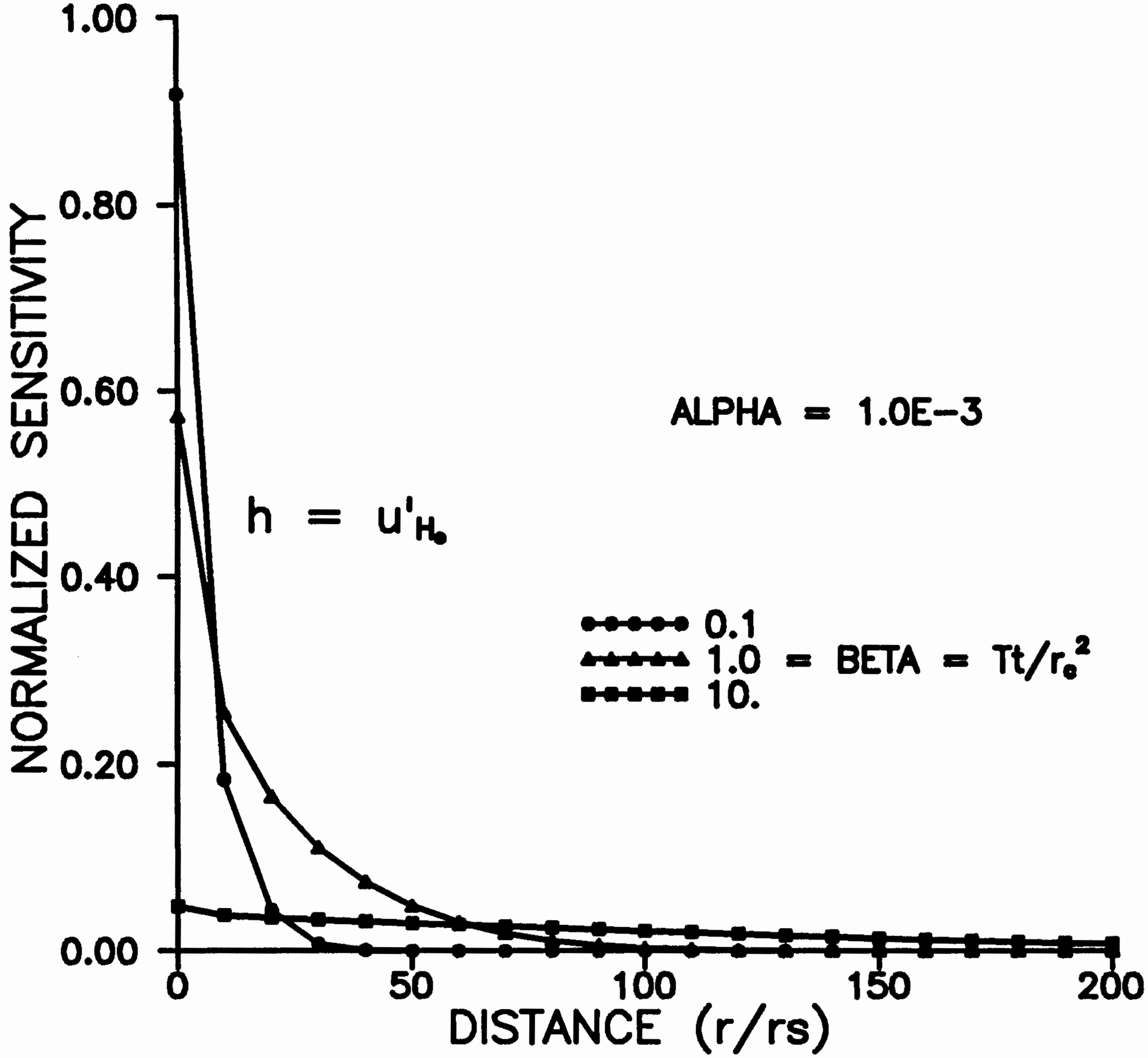
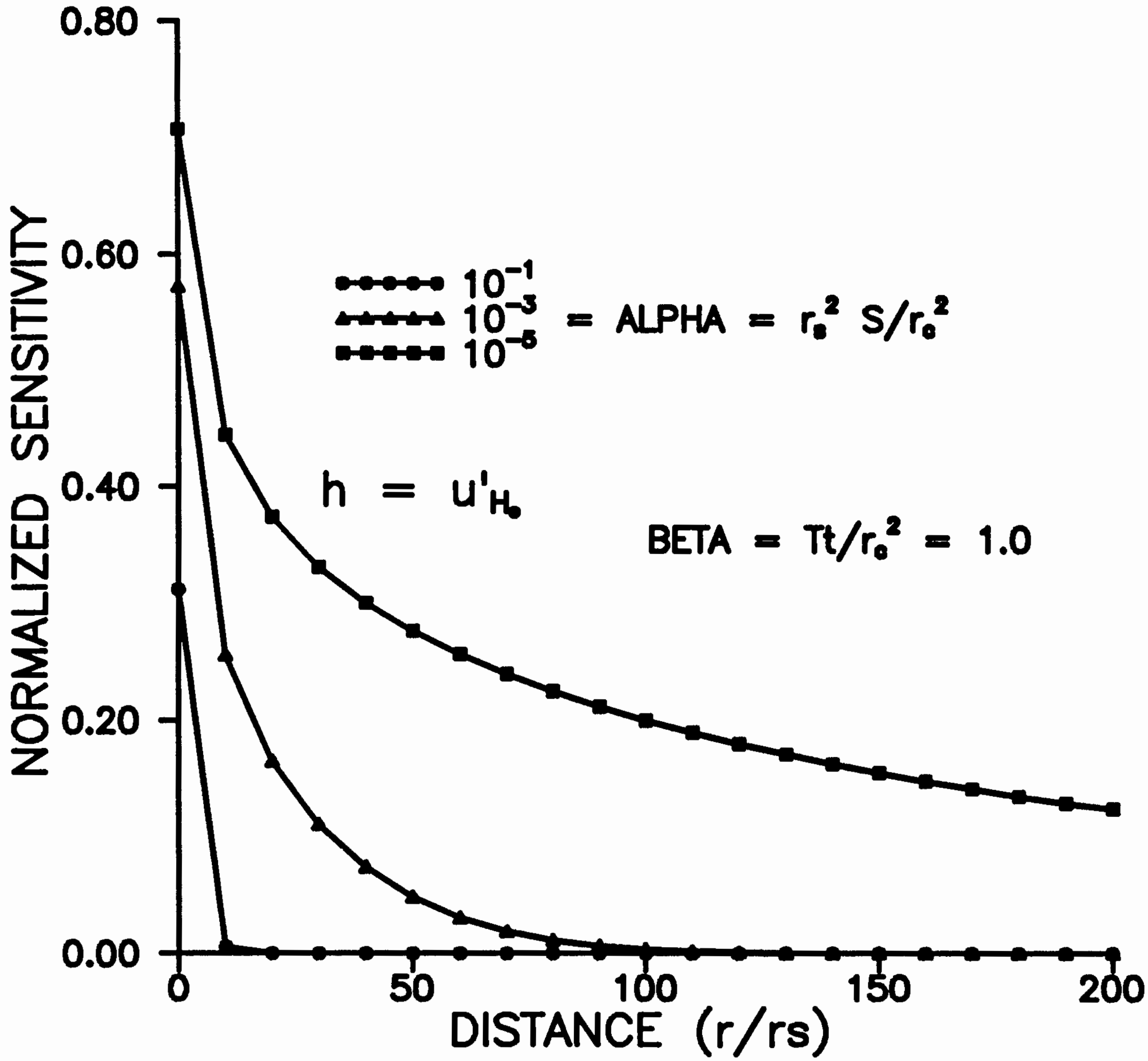


Figure 4.
Variation of u'_{H_o} With Distance For Various Alpha



Sensitivity to Transmissivity

Figures 5-8 illustrate the dependence of the sensitivity to transmissivity on time, distance, and alpha. u'_T has positive and negataive lobes except for $r = r_s$ (Figures 5 and 7). Figure 7 shows that the maximum sensitivity to transmissivity occurs at the well. A given observation well will be sensitive to the transmissivity over a definite time interval (Figure 5), and the sensitivity decays rapidly with increasing r. Figures 6 and 8 illustrate the dependence on the storage coefficient (alpha). The maximum amplitude of the sensitivity seems to vary inversely with alpha (Figure 6), while the amplitude at $r = r_s$ does not seem to have a strong dependence on alpha (Figure 8). As noted previously, we need a smaller alpha (storage coefficient) for sensitivities to propagate farther from the slugged wells. (Figure 8).

Figure 5.
Variation of u'_T With Time For Various r/r_s

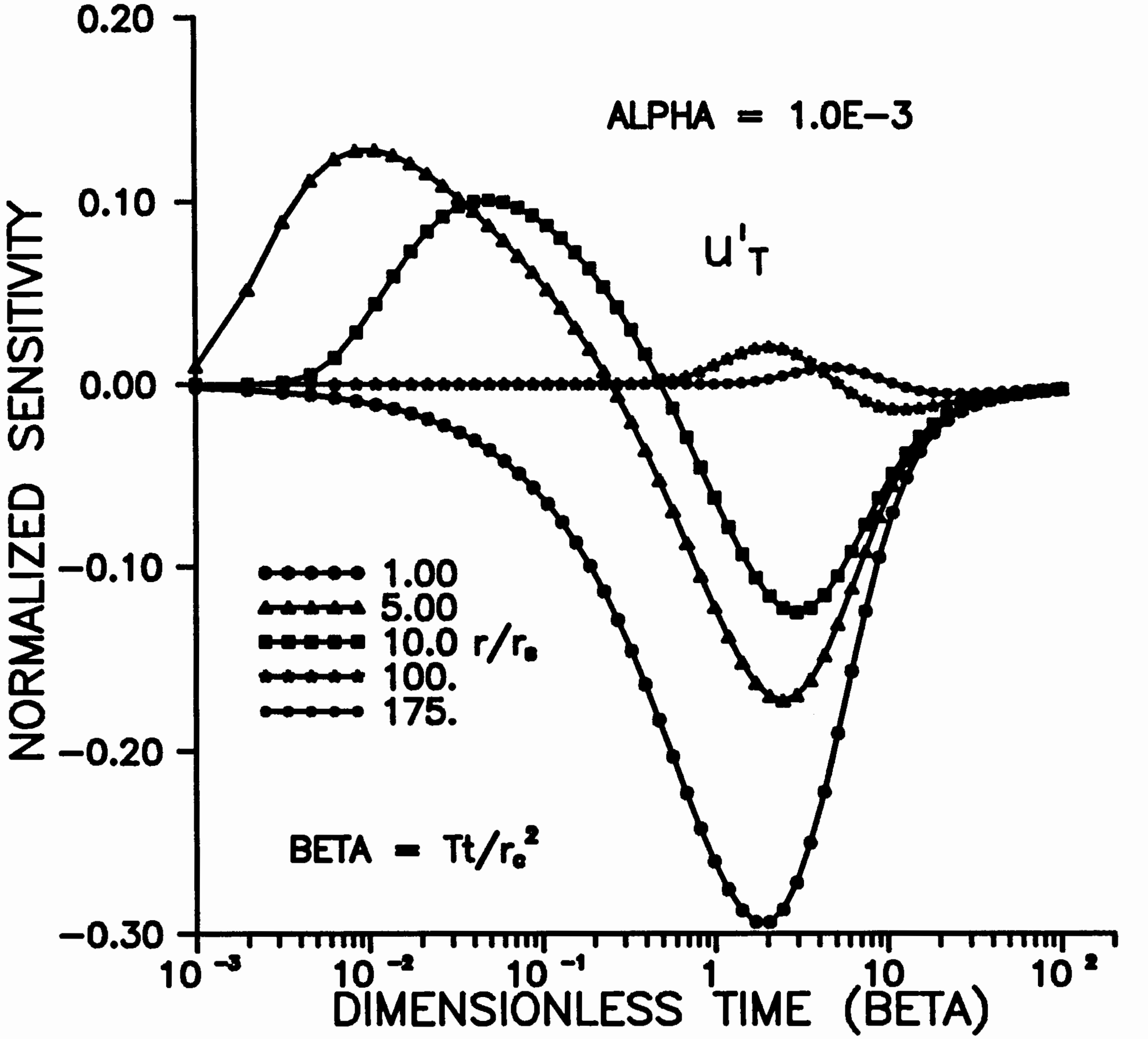


Figure 6.
Variation of u'_T With Time For Various Alpha

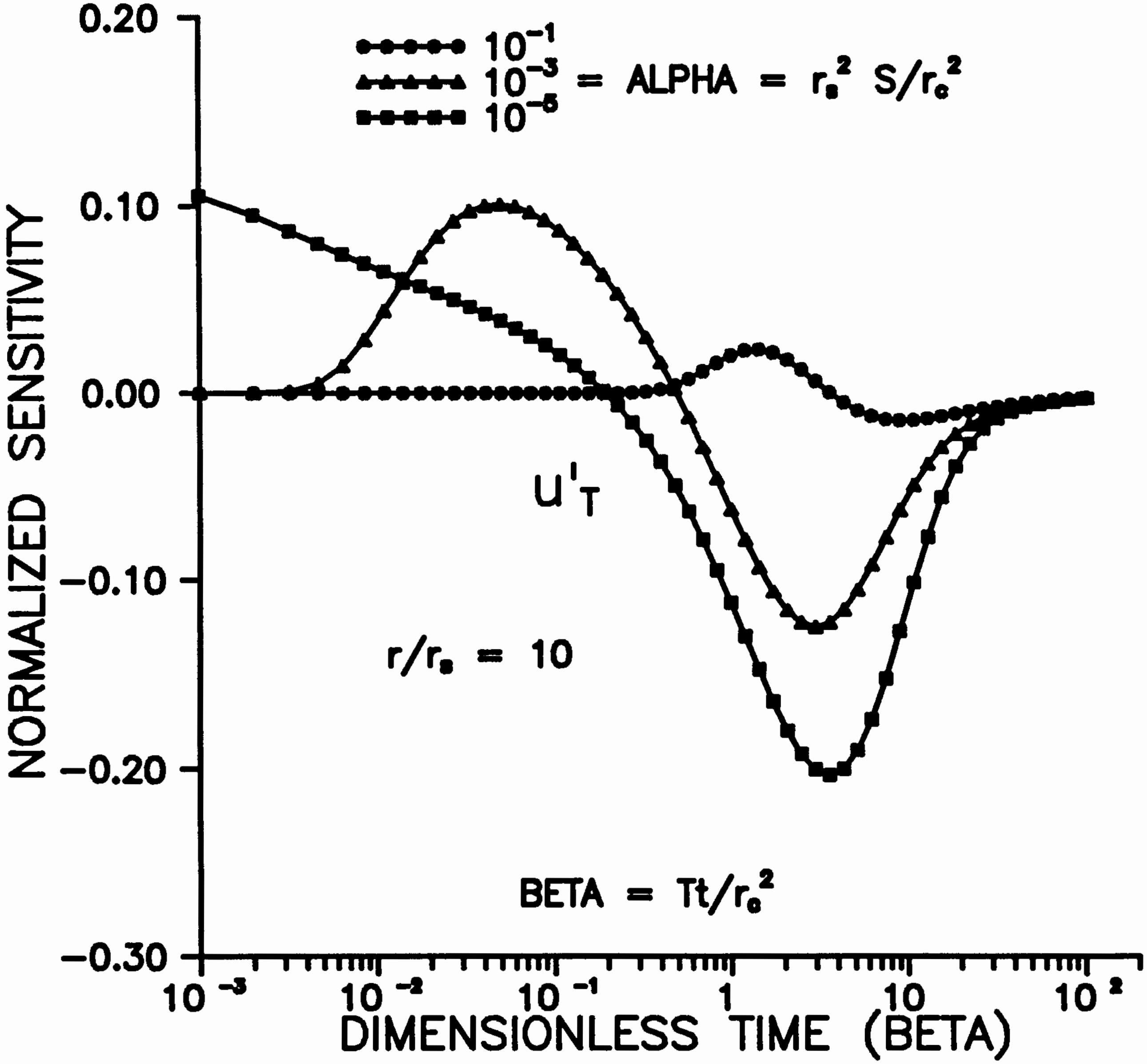


Figure 7.
Variation of u'_T With Distance For Various Times

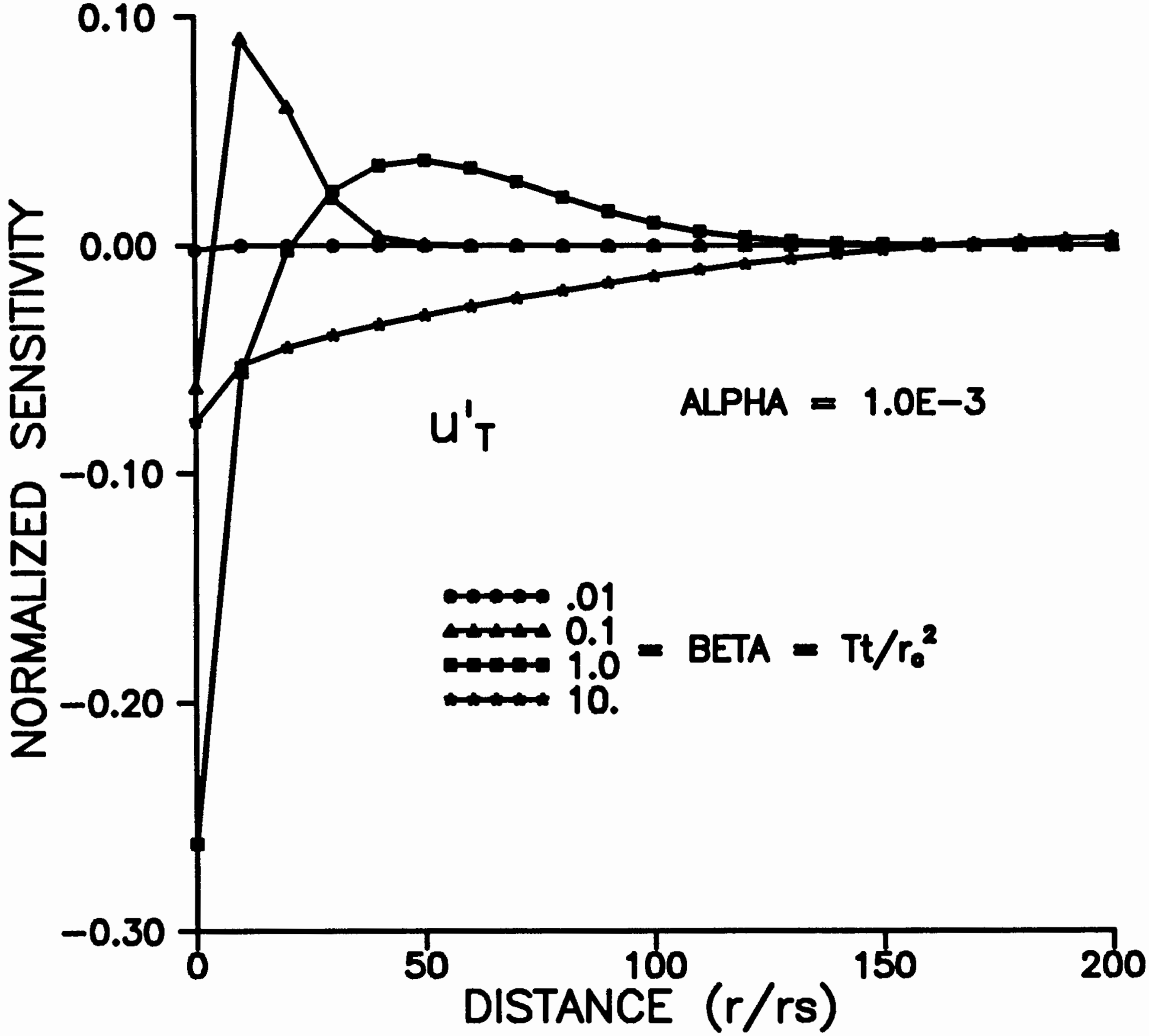
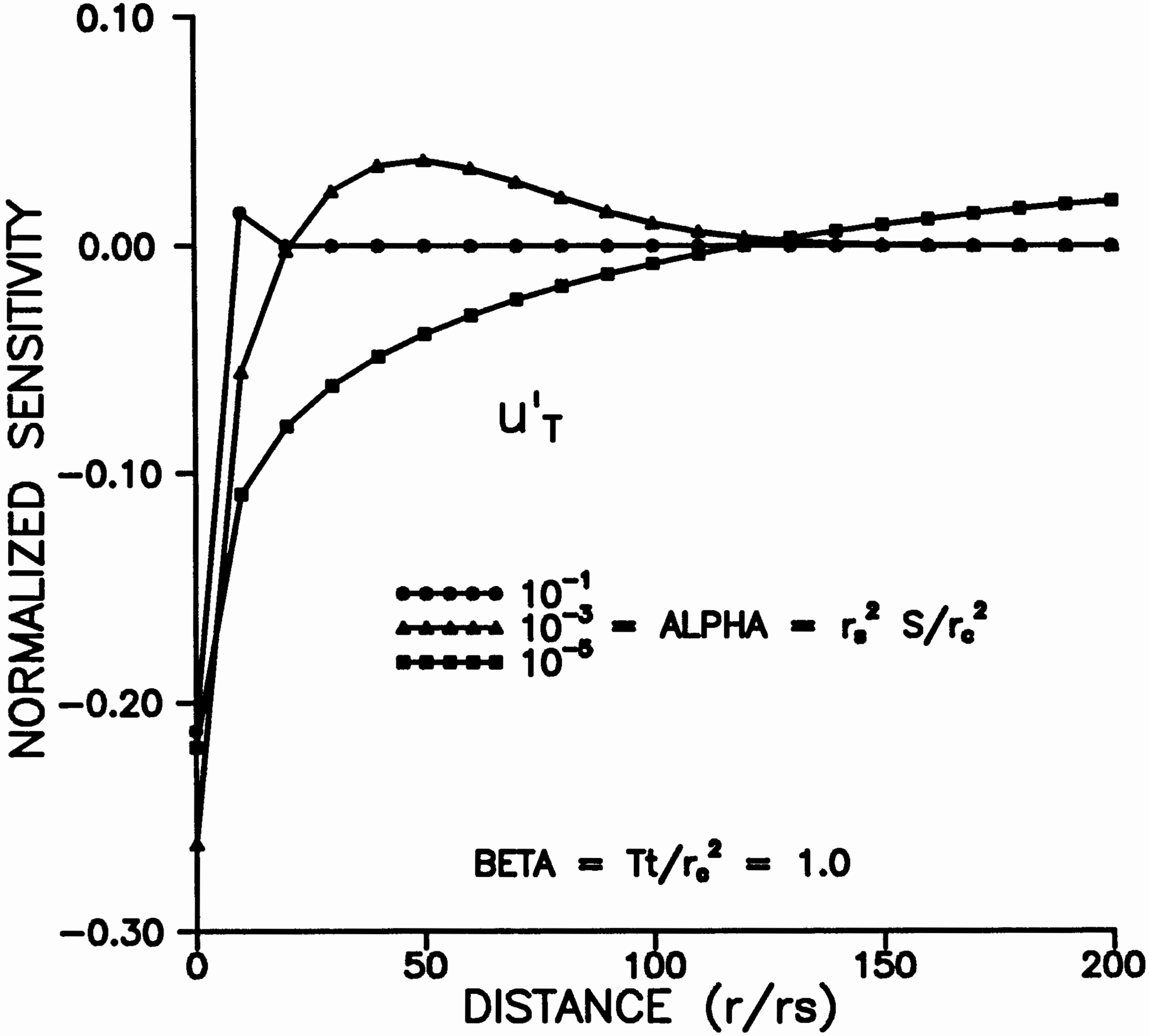


Figure 8.
Variation of u'_T With Distance For Various Alpha



Sensitivity to Storage

Figures 9-12 illustrate the dependence of the sensitivity to storage on α, β, and r. The maximum sensitivity does not occur at $r = r_s$ (Figures 9 and 11), but rather at a distance of about $5r_s$ for $\alpha = 10^{-3}$. Figure 11 shows that the shape of u_s' moves out to larger distances while widening and decaying with increasing time. The dependence on alpha shown in Figure 12 reveals that the signal propagates much farther from the well for smaller values of α. Figure 10 shows that, for a chosen r, the maximum amplitude of the sensitivity is inversely proportional to α and occurs at earlier times for smaller α's.

Figure 9.
Variation of u'_s With Time For Various r/r_s

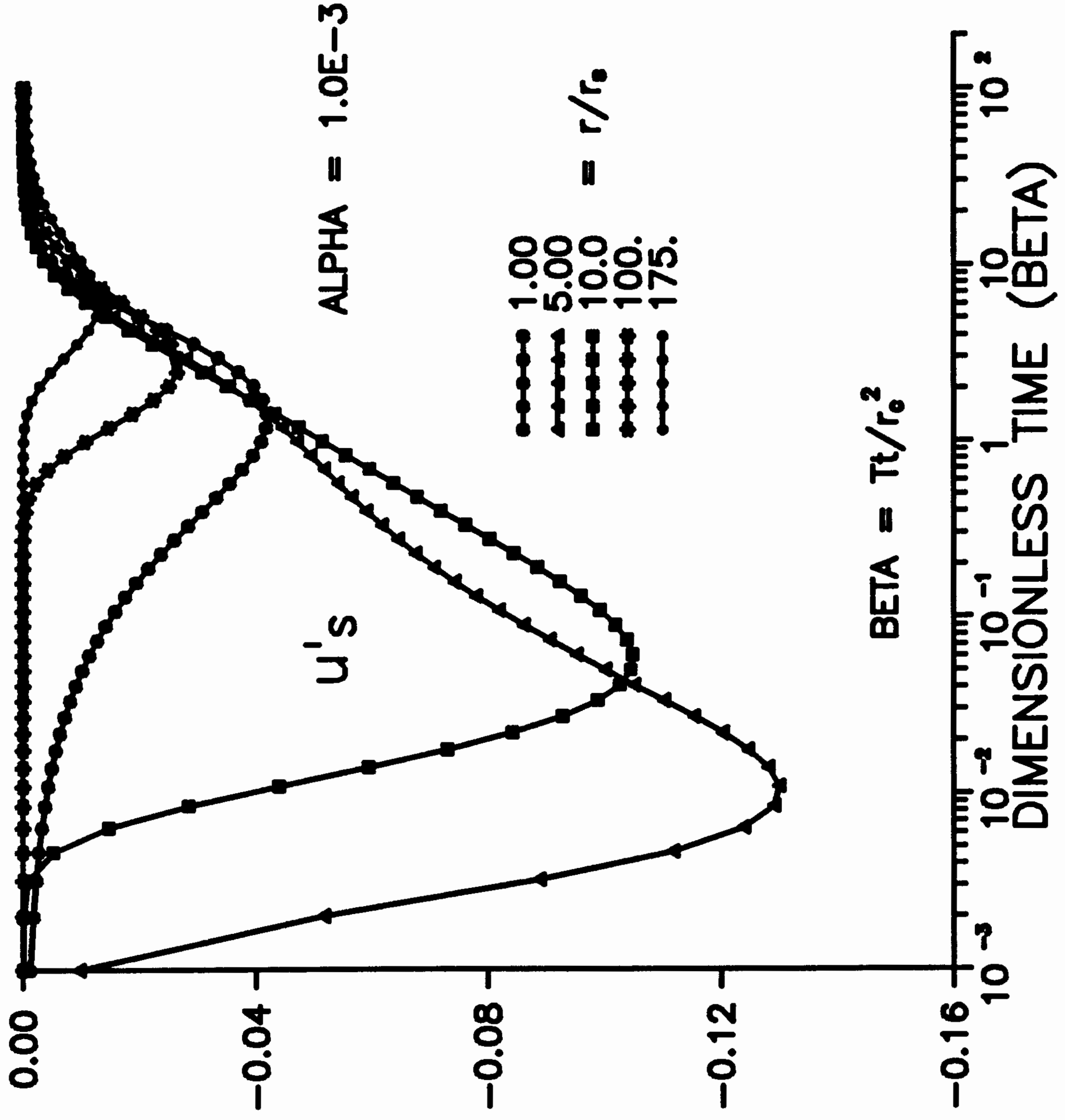


Figure 10.
Variation of u'_s With Time For Various Alpha

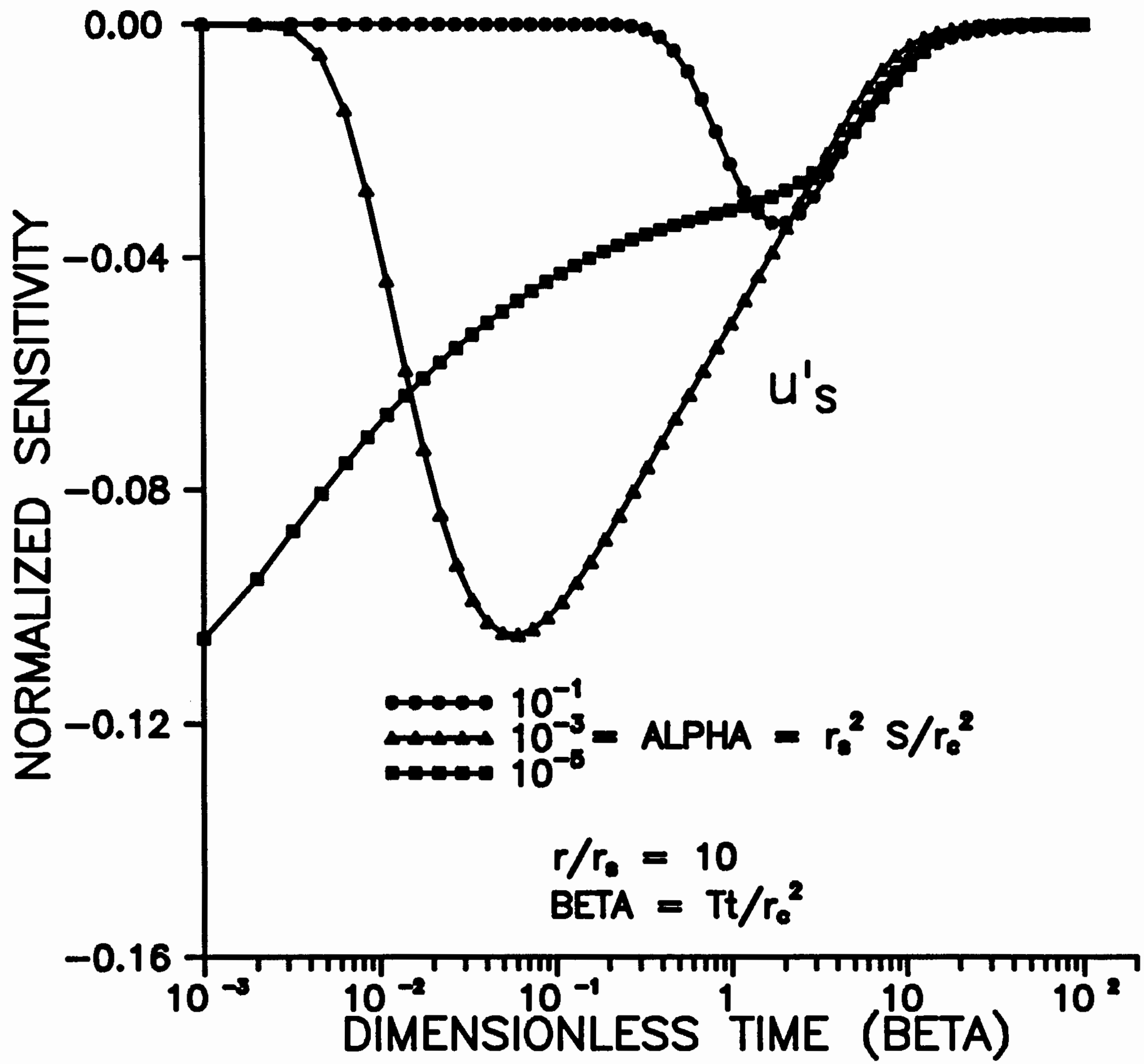
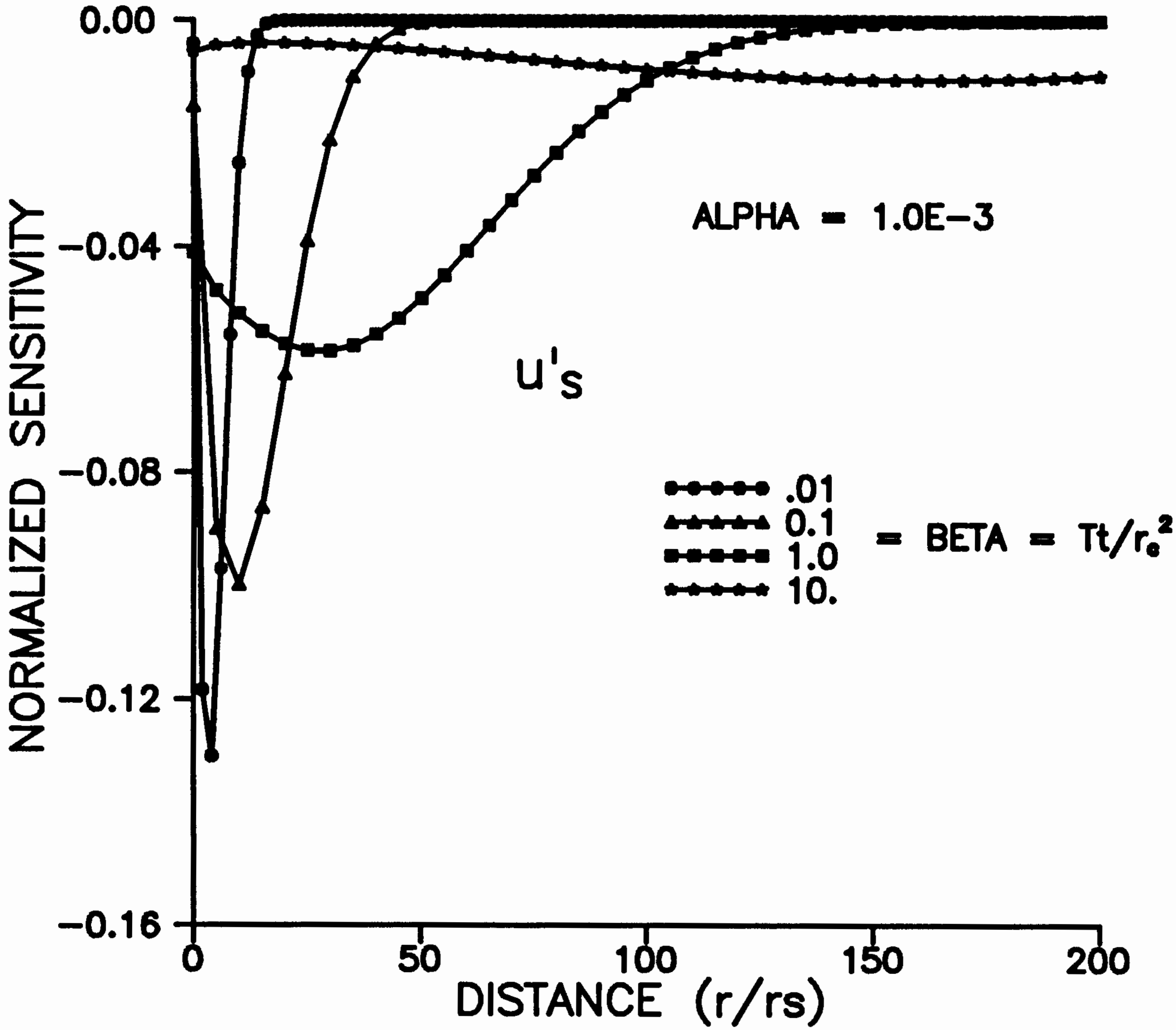


Figure 11.
Variation of u's With Distance For Various Times



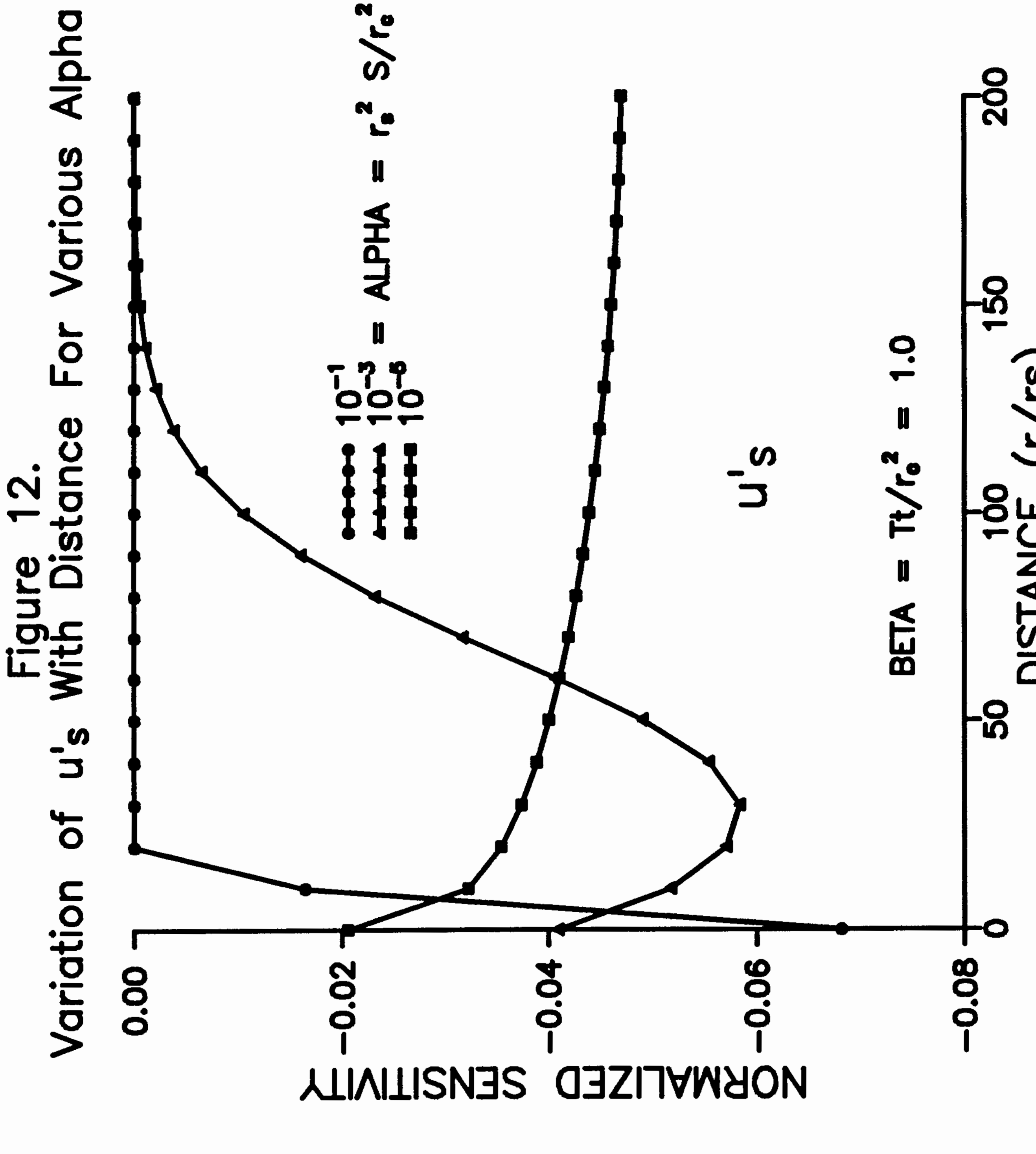


Figure 12.
Variation of u'_s With Distance For Various Alpha
NORMALIZED SENSITIVITY
0.00
-0.02
-0.04
-0.06
-0.08
0
50
100
150
200
DISTANCE (r/rs)
10^{-1}
10^{-3} = ALPHA = r_s^2 S/r_c^2
10^{-5}
u'_s
BETA = Tt/r_c^2 = 1.0

Correlation of u'_T and u'_S

Figure 13 shows that the shape of the sensitivities with respect to transmissivity and storage at $r = r_S$ are extremely similar except for amplitude. This means that data from a slugged well are much more sensitive to T than S and that there will be high correlation between these two parameters. On the other hand, Figure 14 shows the two sensitivities at $r = 10r_S$ and reveals that they have considerably different shapes and nearly the same maximum amplitude. This means that the observation well is much more sensitive to storage and that the correlation between T and S is dramatically reduced by the use of an observation well.

Figure 13.
Comparison of Sensitivities at the Screen Radius

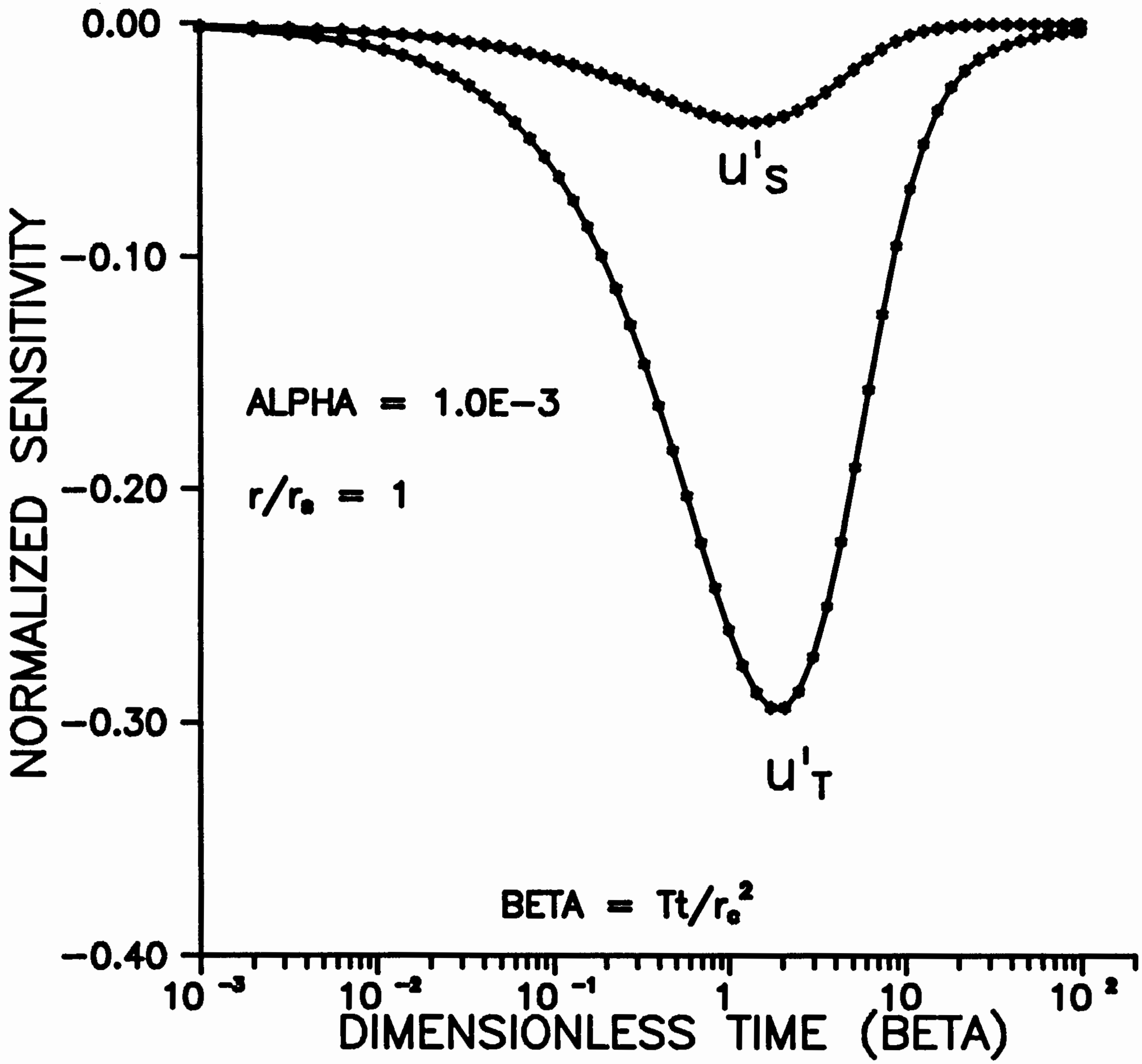
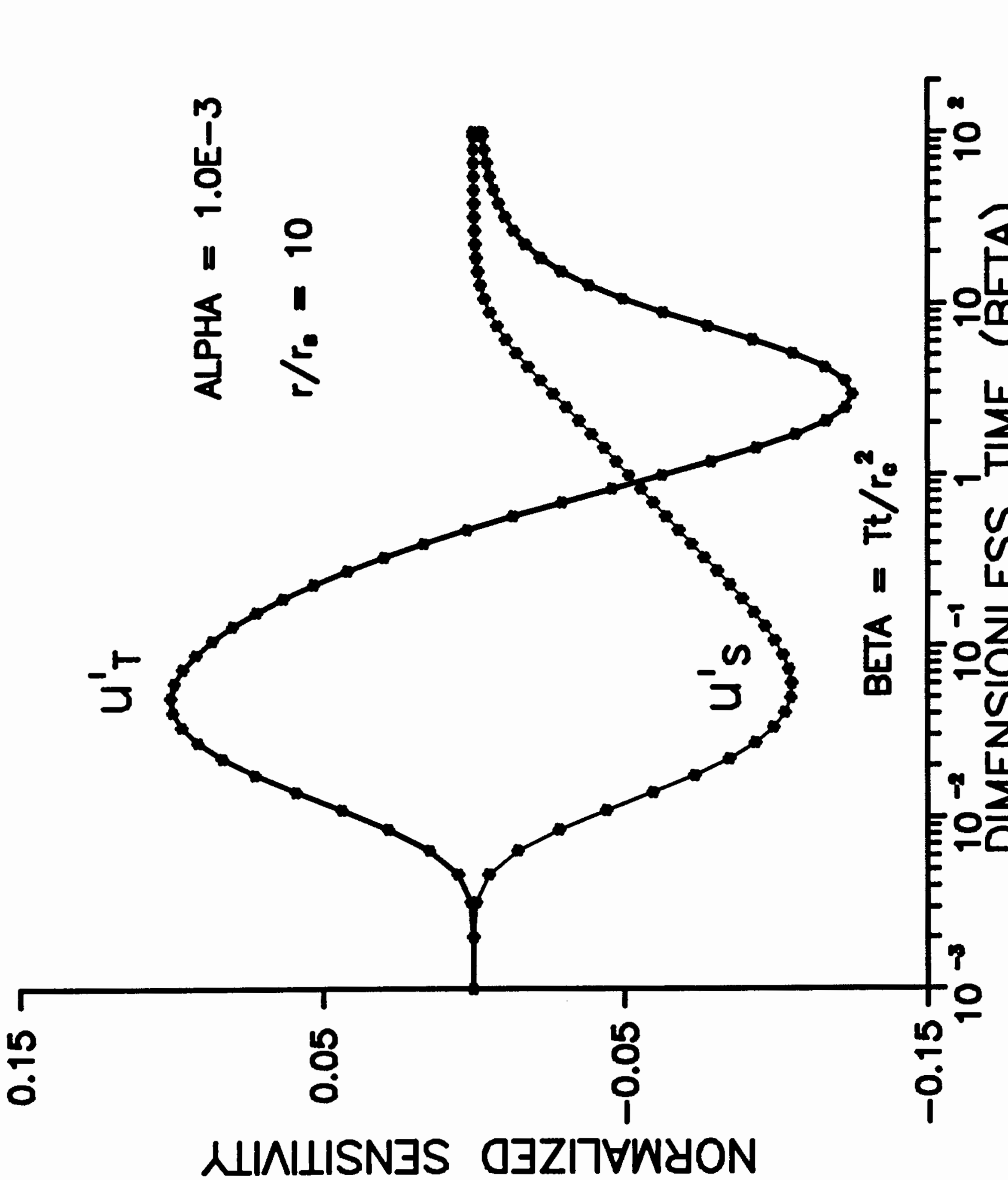


Figure 14.
Comparison of Sensitivities at 10 Screen Radii



Simulation for an Alluvial Field Site

We have developed an alluvial field site for hydraulic testing. It consists of about 35 feet of coarse sand and gravel overlain by about 35 feet of silt and clay. The following is a simulation of expected results at this site. From earlier laboratory work and pumping tests we know some average values for K, T and S.

$$K \approx 300 \text{ ft/day} = .208 \text{ ft/min}$$
$$T \cong (.208 \text{ ft/min})(35 \text{ feet}) = 7.28 \text{ ft}^2\text{/min}$$
$$S \cong .00063$$

We simulate the results for the slugged well and two observation wells at 5 and 10 feet away, taking data over a three minute interval (slug tests are very short duration in this media). It is assumed the slugged well is 4 inches in diameter and all wells are fully screened. The simulated data is rounded to the nearest tenth of a foot and then analyzed in an inverse program.

Results

	Slugged Well Only	**Slugged Well + Obs. Well**	**Slugged Well + 2 Obs. Wells**
Range of T (ft^2./min)	7.11 - 8.00	7.22 - 7.37	7.24 - 7.38
Range of S	$(.178 - .722) \times 10^{-3}$	$(.616 - .666) \times 10^{-3}$	$(.609 - .648) \times 10^{-3}$
Corr.	.98	.54	.44
rms Dev.	.026	.026	.025
Remarks	Trouble Converging	Converged rapidly	Converged Rapidly

Results From Dakota Aquifer, Lincoln County, Kansas

As part of a regional study of the Dakota aquifer in Kansas a number of pumping and slug tests have been performed. One site in Lincoln County was slug tested with an observation well. The following are details of the two wells:

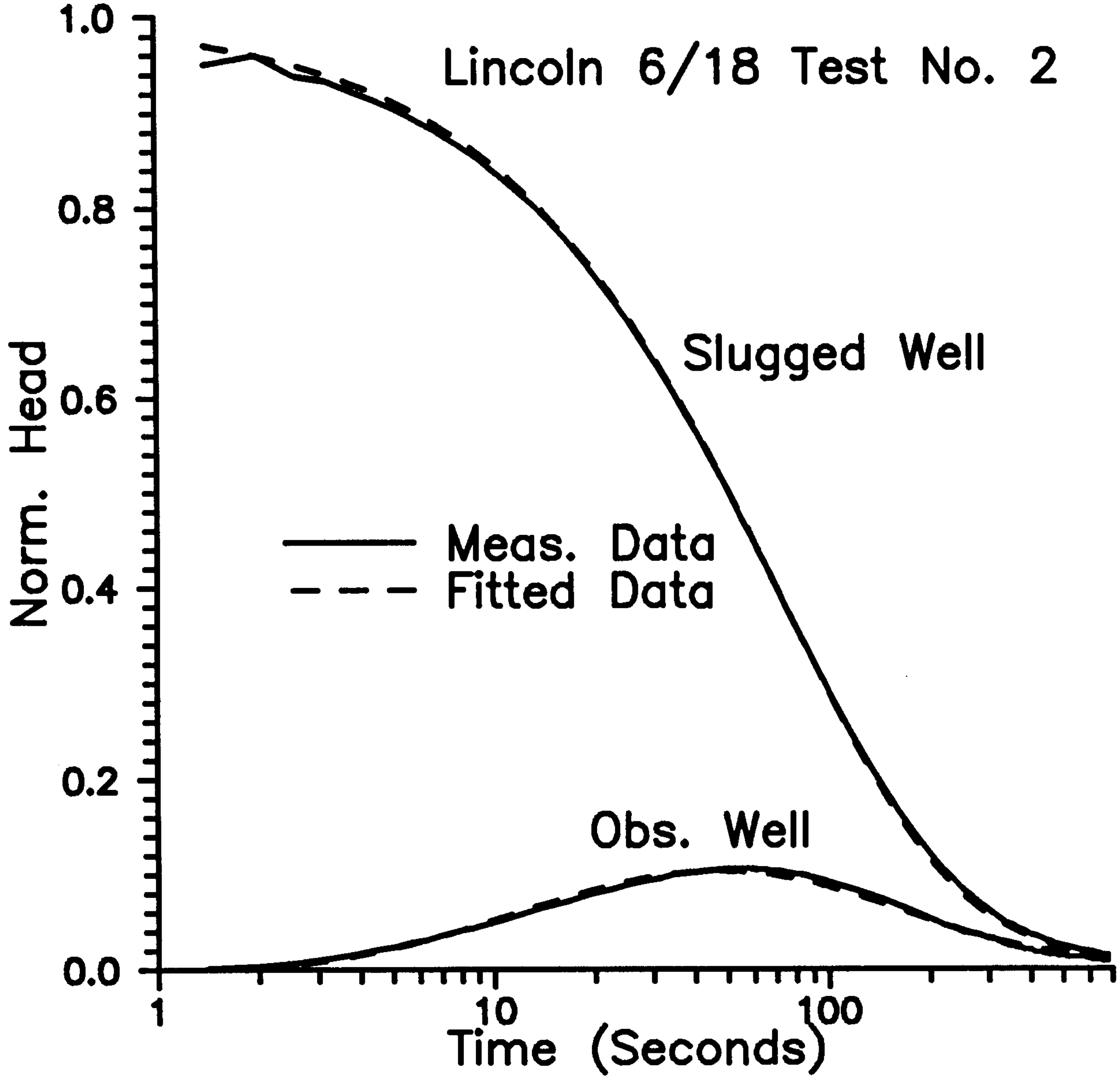
	Slugged Well	Observ. Well
Depth	98.4 feet	94.4 feet
Dia.	4 inches	2 inches
Screen	78-98 feet	84-94 feet

The test was analyzed three ways:

	Slugged Well	**Obser. Well**	**Slugged Well + Obser. Well**
T (ft^2/sec)	$.896 \times 10^{-3}$	1.0005×10^{-2}	1.029×10^{-2}
S	$.2 \times 10^{-3}$	$.514 \times 10^{-4}$	$.520 \times 10^{-4}$
Corr.	.99	.269	.49
rms Dev	.0024	.0030	.0040

It is clear that the use of an observation well greatly improves the estimate for S and makes the inverse problem much better conditioned. Figure 15 shows the field measurements and the fitted data.

Figure 15.
Dakota Aquifer Test, Lincoln County Kansas



Conclusion

While it would usually not be practical to install an observation well solely for use in a slug test, many times nearby wells are available. Generally, the observation well must be fairly close (a few tens of feet or less) to the slugged well to be effective. The storage coefficient must be small in order to see the effect of the slug at greater distances from the slugged well. Since the temporal and spatial dependence of the sensitivities for transmissivity and storage are considerably different, the addition of one or more observation wells will substantially reduce the correlation between these two parameters, and result in much better estimates than usually obtained in slug tests. These ideas have been illustrated using typical data from our research sites.

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