# Sensitivity of Near-Surface Shear-Wave Velocity Determination from Rayleigh and Love Waves

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SUMMARY

Rayleigh and Love waves recorded on seismic-shot gathers can be used to determine the thickness and shear-wave velocity of shallow subsurface layers. After the data are transformed into the k-f domain, the dispersion curve for each of the phases can be picked from maxima on the contour plot. This dispersion curve is then inverted for the velocities and depths. Different frequencies in the dispersion curve yield information about different depths. The fundamental mode has proven to be of greater use than higher modes. Both Rayleigh and Love waves are easily inverted. However, the Love waves seem to yield information in a lower portion of the spectrum than the Rayleigh modes. Three examples are given from field experiments conducted near Canton, Texas.

## INTRODUCTION

Shear-wave velocity of near-surface materials is one of the most important parameters in many geotechnical, hydrological, and other engineering geology application areas. The determination of shear-wave velocity is thus an active area of study. Refraction and reflection are used conventionally to determine the velocity structure. However, in many areas, shallow reflection data quality may be too poor to be of use in determining the velocity structure. In these areas, strong surface waves are usually recorded and can be used in conjunction with the refraction information to determine shear-wave velocities.

Surface-wave information has been utilized by many different investigators. Earthquake seismologists have developed methods using surface waves to investigate the deep crustal and upper-mantle structure of the earth (Schwab and Knopoff, 1972; Braille and Keller, 1975). Furthermore, since 1980 many geophysicists (Cherry et al., 1980; Nazarian and Stokoe, 1986; Barrows and Gahr, 1987; and Mokhtar, 1988) have utilized the ground roll in exploration seismograms for several different applications. This paper discusses some practical considerations which should be understood when using surface waves to determine the shear-wave velocity in shallow-seismic surveys. The forward problem of finding the dispersion curves associated with a particular model was calculated by a FORTRAN program based on the work of Schwab and Knopoff (1972). The inversion problem was solved by the minimum square error method.

## SENSITIVITY OF PARAMETERS

The surface-wave method is based on the analysis of the dispersion curve of the surface wave. For a simple one-layer model, the dispersion curve of the Rayleigh wave is a function of $V_{p1}$, $V_{s1}$, $\rho_1$, $h_1$ of the surface layer and $V_{p2}$, $V_{s2}$, and $\rho_2$ of the half space.

The compressional-wave velocity $V_p$ and density $\rho$ have very little influence on the dispersion curves of the Rayleigh waves. However, shear-wave velocity $V_s$ and layer thickness $h_1$ greatly influence the Rayleigh wave dispersion curve. In other words, $V_s$ and $h_1$ can be determined from the inversion of the Rayleigh-wave dispersion curve, $V_p$ and $\rho$ cannot be found through such an inversion.

In the following sections the sensitivity of the method to changes in the model parameters (those which influence the dispersion curve) will be discussed.

## INFLUENCE OF SURFACE-LAYER THICKNESS

In an area where the velocity structure can be approximated by a simple one-layer/half space model, what influence does surface-layer thickness have on the determination of the half-space velocity? To investigate this question, dispersion curves for several one-layer models (Table 1) were generated for two different half-space velocities. The difference in half-space velocities was approximately 20 percent.

The inversion procedure can easily distinguish between two models with different dispersion curves as long as the apparent velocity values differ by at least 5 percent at any given frequency. Thus, the 20 percent difference in half-space velocity used in the simple one-layer models can only be detected using Rayleigh waves in the portion of the spectrum where their dispersion curves differ in apparent velocity by at least 5 percent.

Using an initial surface-layer thickness (Table 1) of 5 feet, the dispersion curves differed by 5 percent at frequencies of 120 Hz and below. Using a surface-layer thickness of 10 feet, the upper limit of useful Rayleigh-wave frequencies is 57.5 Hz (Table 1). In general, the results shown in Table 1 indicate that as the surface layer becomes thicker, only the longer wavelength Rayleigh waves are sensitive to changes in the shear-wave velocity of the half-space $V_{s2}$.

In the set of models utilized to generate Table 1, it should be obvious that the frequencies sensitive to changes in $V_{s2}$ are approximately twice the depth to the half space in wavelength. This indicates that it should be possible to develop simple rules-of-thumb for use in designing surface-wave experiments in new field areas, given some simple information concerning the expected velocity structure.

## MULTIPLE LAYERS

In areas where the velocity structure is more complicated, how do layer thicknesses and shear-wave velocities affect the inversion results? In this section a model consisting of two layers and a half space will be used to investigate this question. The basic parameters are $V_{p1}$ = 1800 ft/s; $V_{p2}$ = 3500 ft/s; $V_{p3}$ = 7000 ft/s; $V_{s1}$ = 650 ft/s; $V_{s2}$ = 1200 ft/s; $V_{s3}$ = 2300 ft/s; $\rho_1$ = 1.8 g/cm$^3$; $\rho_2$ = 1.9 g/cm$^3$; $\rho_3$ = g/cm$^3$; $h_1$ = 10 ft; $h_2$ = 5 ft. When the second layer thickness, $h_2$, is changed from 5 ft to 40 ft (see Table 2) and when $V_{s2}$ is changed, phase velocities change only in the middle of the dispersion curves. Comparing them to the last section, it is seen that changing the shear-wave velocity of the surface layer, $V_{s1}$, only influences the high-frequency part of the dispersion curves, changing the shear-wave velocity of the middle layer, $V_{s2}$, only influences the middle frequency part of dispersion curves, and changing the shear-wave velocity of the half space, $V_{s3}$, only influences the low-frequency part of the dispersion curves.

As the middle layer increases in thickness, the bandwidth of the useful portion of the spectrum increases (Table 2). This means that it should be easier to invert for the velocity of a thick middle unit than a thin middle unit since a larger portion of the dispersion curve should contain useful information.

## COMPARISON OF LOVE WAVES WITH RAYLEIGH WAVES

When a horizontal source and horizontal geophones are used to impart and detect ground motion normal to the direction of the line, the corresponding ground roll consists of Love waves. For a simple one-layer case, the Love-wave dispersion curves are sensitive only to changes in $V_{s1}$, $V_{s2}$ and $h_1$. This is similar to the behavior of the Rayleigh-wave dispersion curves. Many of the other observations made about Rayleigh-wave dispersion curves hold for the Love-wave case, also.

For example, using the one-layer model, when thickness $h_1$ changes from 5 feet to 600 feet, the cut-off of useful dispersion frequencies gradually changes from 75.9 Hz to 0.63 Hz (Table 3). The main difference in the use of Love waves versus Rayleigh waves lies in the wavelengths needed to penetrate to a certain depth. Comparing Table 3 with Table 1, the wavelengths necessary to recover velocity information from any given depth are almost twice as long for Love waves than they are for Rayleigh waves. This also means, however, that Love waves are more sensitive to shallow geology at any given wavelength.

## COMPARISON OF THE SECOND MODE WITH THE FUNDAMENTAL MODE

The dispersion curve of the second Rayleigh mode is similar to the curve for the primary mode. However, as the depth of the target layer is increased, the sensitivity of the secondary dispersion curve falls off much more rapidly than the primary curve. This means that the second-mode Rayleigh dispersion curve is only useful for determining $V_s$ for shallow layers.

Using the second mode also requires more care than using the fundamental mode because the second mode of the dispersion curves is higher than the fundamental mode in both phase velocity and frequency. After transferring the seismic data to the k-f domain, the second mode dispersion curve is more difficult to pick than the fundamental mode dispersion curve because the curve is easily confused with body waves in that domain.

Similar conclusions can be obtained about secondary-mode Love waves. The sensitivity of the secondary-mode Love waves decreases even more quickly with depth than the secondary-mode Rayleigh waves. This means that the secondary-mode Love waves can also only be used to determine $V_s$ for shallow layers.

## EXAMPLES

The following are three examples from the area around Canton, Texas. The compressive-wave velocities and thicknesses are known from refraction: $V_{p1}$ = 1212 ft/s; $V_{p2}$ = 3333 ft/s; $V_{p3}$ = 5333 ft/s; $V_{p4}$ = 6000 ft/s; $V_{p5}$ = 6225 ft/s; $h_1$ = 5 ft; $h_2$ = 12 ft; $h_3$ = 30 ft; $h_4$ = 19 ft. The shear-wave velocity information is known from shear-wave refraction and log information: $V_{s1}$ = 544 ft/s; $V_{s2}$ = 656 ft/s; $V_{s3}$ = 1600 ft/s; $V_{s4}$ = 1867 ft/s (by refraction); $V_{s5}$ = 2500 ft/s (log).

Example 1: A rifle as a source. P-wave geophones (40 Hz), $\Delta X$ = 2 ft, Xo = 2 ft; $\Delta t$ = 1 ms were utilized. There are 84 recorded traces shown in Figure 1, and the fundamental Rayleigh-wave dispersion curve is shown in Figure 2. The inversion results are $V_{s1}$ = 467 ft/s; $V_{s2}$ = 656 ft/s; $V_{s3}$ = 1167 ft/s; $V_{s4}$ = 1350 ft/s; $V_{s5}$ = 1751 ft/s, and $h_1$ = 3.6 ft; $h_2$ = 15.3 ft; $h_3$ = 24 ft; $h_4$ = 14 ft.

Example 2: A rifle as a source. P-wave geophones (10 Hz), $\Delta X$ = 30 ft, $X_o$ = 60 ft, $\Delta t$ = 2 ms were utilized. There are 24 recorded traces shown in Figure 3, and the fundamental Rayleigh-wave dispersion curve is shown in Figure 4. Its inversion results are $V_{s1}$ = 933 ft/s; $V_{s2}$ = 741 ft/s; $V_{s3}$ = 1167 ft/s; $V_{s4}$ = 1350 ft/s; $V_{s5}$ = 2451 ft/s; and $h_1$ = 6.6 ft, $h_2$ = 12.8 ft, $h_3$ = 26.5 ft and $h_4$ = 23 ft.

Comparing the surface-wave results with the velocity structure known from refraction and downhole logging information, it is seen that the high-frequency information from example 1 accurately inverts for the shallow-velocity structure, while the lower-frequency information from example 2 inverts more accurately for the deeper structure.

Example 3: Sledge hammer as a source. S-wave geophones (4 Hz), $\Delta X$ = 30 ft; $X_o$ = 30 ft; $\Delta t$ = 2 ms were utilized. There are 24 recorded traces shown in Figure 5 and the fundamental Love-wave dispersion curve is shown in Figure 6. Its inversion results are $V_{s1}$ = 452 ft/s; $V_{s2}$ = 589 ft/s; $V_{s3}$ = 1389 ft/s; $V_{s4}$ = 2429 ft/s; $V_{s5}$ = 2456 ft/s; and $h_1$ = 3.4 ft, $h_2$ = 14.4 ft, $h_3$ = 34.4 ft and $h_4$ = 30.8 ft. Compared with refraction and log information, these results seem reasonable. In fact, the refraction and surface-wave information within a record can be used together (see

Figures 1, 3, and 5) to obtain more information about velocity.

## CONCLUSIONS

Using surface waves (Rayleigh and Love), it is possible to determine shear-wave velocity and thickness of shallow layers, but it is difficult to determine $V_p$ and ρ. Different frequencies of the dispersion curves respond at different depths. This is noticed in both field work and inversion work.

Generally, using the fundamental mode of the dispersion curve is better than using higher modes of the dispersion curve. Using surface waves to determine $V_s$ and h of shallow layers is a useful and economical method. We believe that by combining P-wave refraction, Rayleigh wave, S-wave refraction, and Love-wave information to investigate velocity, better velocity information can be obtained from data records in a variety of environments.

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**Table 1. Usable frequency cutoff vs. layer thickness, $h_1$ (Rayleigh waves).**

| H1 (ft) | F (Hz) | C1 (ft/s) | C2 (ft/s) | (C2-C1)/C1 (%) | λ (ft) |
|---|---|---|---|---|---|
| 5 | 120 | 1391 | 1461 | 5.0 | 11.6 |
| 10 | 57.5 | 1444 | 1554 | 7.6 | 25.1 |
| 20 | 30.2 | 1384 | 1457 | 5.3 | 45.8 |
| 40 | 14.5 | 1439 | 1546 | 7.4 | 99.6 |
| 80 | 7.6 | 1379 | 1445 | 4.9 | 182 |
| 160 | 3.8 | 1379 | 1445 | 4.9 | 364 |
| 300 | 2.0 | 1396 | 1474 | 5.6 | 702 |
| 600 | 0.96 | 1448 | 1562 | 7.9 | 1513 |

**Table 2. Usable frequency cutoff vs. layer thickness, $h_2$ (Rayleigh waves).**

| H1 (ft.) | H2 (ft.) | F (Hz.) | C1 (ft/s) | C2 (ft/s) | (C2-C1)/C1 (%) | λ (ft.) |
|---|---|---|---|---|---|---|
| 10 | 5 | 30.2 | 841 | 910 | 8.3 | 28 |
| | | 27.5 | 1047 | 1200 | 14.6 | 38 |
| | | 25.1 | 1306 | 1399 | 7.1 | 52 |
| 10 | 10 | 30.2 | 802 | 875 | 9.1 | 27 |
| | | 25.1 | 1113 | 1309 | 17.6 | 44 |
| | | 22.9 | 1353 | 1459 | 7.8 | 59 |
| 10 | 30 | 30.2 | 791 | 863 | 9.1 | 26 |
| | | 17.4 | 1150 | 1433 | 24.6 | 66 |
| | | 12.0 | 1737 | 1830 | 5.4 | 144 |
| 10 | 40 | 30.2 | 863 | 791 | 9.1 | 26 |
| | | 13.2 | 1215 | 1509 | 24.2 | 92 |
| | | 10.0 | 1658 | 1822 | 9.9 | 166 |

**Table 3. Usable frequency cutoff vs. layer thickness, $h_1$ (Love waves).**

| H1 (ft) | F (Hz) | C1 (ft/s) | C2 (ft/s) | (C2-C1)/C1 (%) | λ (ft) |
|---|---|---|---|---|---|
| 5 | 75.9 | 1571 | 1650 | 5.0 | 20.7 |
| 10 | 36.7 | 1595 | 1686 | 5.7 | 43.4 |
| 20 | 18.9 | 1572 | 1652 | 5.1 | 83.0 |
| 40 | 9.57 | 1563 | 1640 | 4.9 | 163 |
| 80 | 4.79 | 1563 | 1640 | 4.9 | 327 |
| 160 | 2.39 | 1563 | 1640 | 4.9 | 653 |
| 300 | 1.20 | 1613 | 1714 | 6.2 | 1349 |
| 600 | 0.63 | 1572 | 1652 | 5.1 | 2491 |

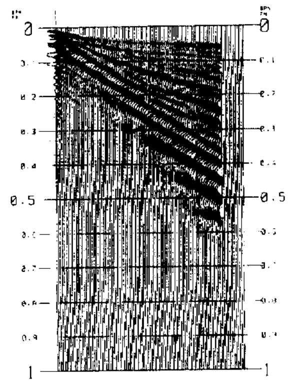


FIG. 1. P-wave shot record (40-Hz phones, $\Delta X = 2$ ft, $X_o = 2$ ft; $\Delta t = 1$ ms).

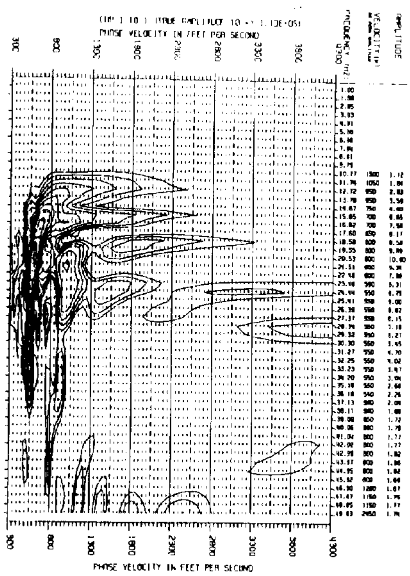

FIG. 2. Dispersion contour plot based on Fig. 1.

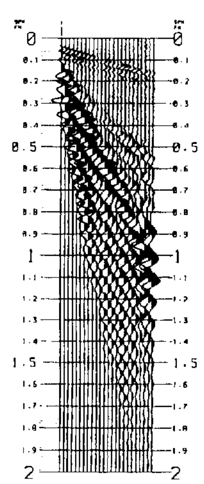

FIG. 3. P-wave shot record (10-Hz phones, $\Delta X$ = 30 ft, $X_0$ = 60 ft, $\Delta t$ = 2 ms).

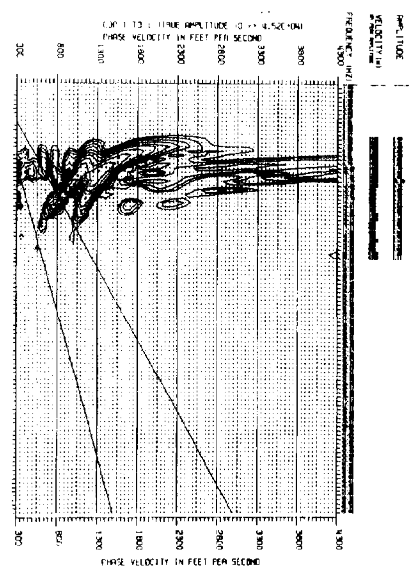

FIG. 4. Dispersion contour plot based on Fig. 3.

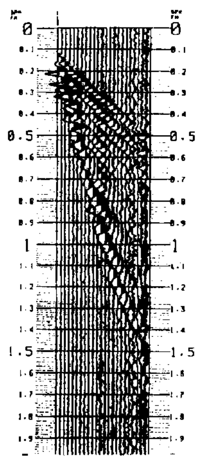

FIG. 5. S-wave shot record (4-Hz phones, $\Delta X$ = 30 ft; $X_0$ = 30 ft; $\Delta t$ = 2 ms).

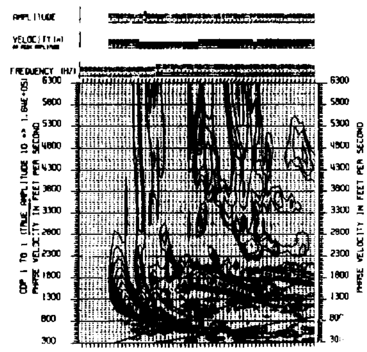

FIG. 6. Dispersion contour plot based on Fig. 5.